Crossing and Radon Tomography for Generalized Parton Distributions

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Abstract

The crossing properties of the matrix elements of non-local operators, parameterized by Generalized Parton Distribution, are considered. They are especially simple in terms of the Double Distributions which are common for the various kinematical regions. As a result, Double Distributions may be in principle extracted from the combined data in these regions by making use of the inverse Radon transform, known as a standard method in tomography. The ambiguities analogous to the ones for the vetor potential in the two-dimensional magneto-statics are outlined. The possible generalizations are discussed.

Introduction

Deeply Virtual Compton Scattering (DVCS) [1, 2] is the cleanest hard process which is sensitive to the Generalized Parton Distributions (GPD), the most popular version of which is probably represented by the Skewed Parton Distributions (SPD), and has been the subject of extensive theoretical investigations for a few years. First experimental data became recently available (see e.g. [3, 4]) and much more data are expected from JLAB, DESY, and CERN in the near future. In the present note we shall limit ourself to the case of the pion target, although our consideration can be applicable to any target.

The crossing version is then provided by the process $\gamma^*\gamma \to \pi\bar{\pi}$ with a highly virtual photon but small hadronic invariant mass W. It was recently investigated in the framework of QCD factorization[5, 6, 7]. It allows to study the pion pair produced in the isoscalar channel, where the huge ρ-meson peak is absent. This process is analogous to the single pion production, described by the pion transition form-factor, being the long time object of QCD studies [8, 9]. In particular, the generalized distribution amplitude (GDA), describing the non-perturbative stage of this process, is a natural counterpart of the pion light cone distribution amplitude. From the other side, it may be considered as a crossing counterpart of pion SPD [5] and related to it [10] by making use of the suitable polynomial basis.

The alternative (and quite elegant) description of non-perturbative stage of DVCS and other hard exclusive processes is provided by Double Distributions (DD) [1, 11]. They

naturally explain the polynomiality of SPD and can be a good starting point for the model-building. At the same time, it is not clear how one can express DD in terms of the hadronic matrix element and extract them, at least in principle, from the data.

In this note we study the crossing properties of the double distributions. We conclude, that both mentioned channels may be described by the common DD, while SPD and GDA may be obtained by its integration over the straight lines constituting the various angles with the coordinate axes. This would allow us to recover the DD from the SPD and GDA by making use of the inverse Radon transform - the mathematical tool widely used for computer tomography. We discuss the ambiguities resulting from the presence of the so-called Polyakov-Weiss (PW) terms in DD and find their close analogy with that for the recovering of the vector potential from the magnetic field. We also outline the possible applications of the method and its further development.

Double distributions in the crossing related channels

Let us start with the following symmetric representation of the matrix elements of twist-2 non-local operators (c.f. [10])

$$
\langle p'|\bar{\psi}\left(-\frac{z}{2}\right)\gamma \cdot z\psi\left(\frac{z}{2}\right)|p\rangle = (2P \cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dye^{-ixPz-iy\Delta z/2}F(x,y,\Delta^2) \tag{1}
$$

$$
+(\Delta \cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dye^{-ixPz-iy\Delta z/2}G(x,y,\Delta^2);
$$

$$
\langle p',-p|\bar{\psi}\left(-\frac{z}{2}\right)\gamma \cdot z\psi\left(\frac{z}{2}n\right)|0\rangle = (2P \cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dye^{-ixPz-iy\Delta z/2}f(x,y,\Delta^2)
$$

$$
+(\Delta \cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dye^{-ixPz-iy\Delta z/2}g(x,y,\Delta^2);
$$

where we (in order to express the crossing properties in the most simple way) adopted the common notations for both channels: $P = (p + p')/2, \Delta = p' - p = 2\xi P + \Delta_\perp$, while p, p' are the initial and final momenta of pion in the DVCS channel, respectively. In the $\gamma^*\gamma \to \pi\bar{\pi}$ channel, Δ corresponds to the total momentum of pion pair while $2P$ is the relative momentum of pions.

The skewedness parameter varies in the region $0 < \xi < 1$ for the DVCS channel and $|\xi| > 1$ for the $\gamma^*\gamma \to \pi\bar{\pi}$ channel, respectively. In what follows we restrict ourselves to the leading twist level, neglecting the transverse momentum $\Delta_\perp$.

The crossing symmetry states that in the different channels the matrix elements are described by the common analytical functions, so that one may relate them by analytical continuation. One should note the crucial advantage of the DD for the studies of crossing properties, namely, that ξ does not enter explicitly to their definition. The only variable, which should be continued for DD is therefore Δ^2 which is equal to t in the DVCS channel and to the squared invariant mass of the pion pair W^2 in the $\gamma^*\gamma \to \pi\bar{\pi}$ channel. The dependence on these variables is entirely non-perturbative and is a subject of separate investigation. This dependence is beyond the scope of our studies, and we shall put both t and W^2 to zero. This point is the common unphysical point for the both channels, so one may conclude, that

$$F(x, y, 0)) = f(x, y, 0); \ G(x, y, 0) = g(x, y, 0) \tag{2}$$

The Skewed Parton Distribution are related to the Double ones by the integration over straight lines [11]. The symmetric skewed distribution is related to symmetric double distributions F and G by the formula, being the straightforward generalization of [11].

$$H(z, \xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y). \tag{3}$$

In turn, the Generalized Distribution Amplitudes, describing the non-perturbative stage of hadron pair production, may be related to the respective Double Distributions in an analogous way:

$$\Phi(t, 1/\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (g(x, y) + \frac{f(x, y)}{\xi}) \delta(t - x/\xi - y) \tag{4}$$

Note the difference with the original definition of GDA [5], where skewedness was defined by the fraction of the pair momentum carried by the one of the produced pions, while here it is defined (in the more symmetric way) by the difference of their momenta. As a result, the analytic continuation between Φ and H is provided by the formula:

$$H(z, \xi) = sign(\xi) \Phi(\frac{z}{\xi}, \frac{1}{\xi}) \tag{5}$$

Contrary to (2), SPD and GDA, which are depending on ξ explicitly, correspond to the different regions of this variable: $\xi < 1$ for SPD and $|\xi| > 1$ for GDA. In what follows, we shall consider SPD in the 'extended' region, on the whole real axis, assuming that for $|\xi| > 1$ it should be substituted by GDA from (5), while the region $-1 < \xi < 0$ is accessible by the use of symmetries.

These discrete symmetries of the SPD and GDA may be now described in the unified way. Namely, the symmetries of DD with respect to its second argument

$$F(x, y) = F(x, -y) \ ; \ G(x, y) = -G(x, -y), \tag{6}$$

correspond to either T-invariance for SPD, or charge conjugation invariance for GDA. This is not surprising, as the T-invariance, first studied in the case of the forward twist-3 matrix elements [12], corresponds to the interchange of both hadrons and partons, in complete analogy with charge conjugation. Note that the reality of the scalar function (SPD or DD) plays the crucial role in such a derivation. At the same time, GDA may acquire the non-trivial imaginary phases due to cut in W^2. However, it is only the real part of GDA which is described by crossing (2), while the imaginary parts requires the separate study of the analytical continuation in W^2. Also, as soon as the chosen definition of GPD preserves P - invariance, such a similarity of T- and C- invariance is related to CPT-theorem. Note that the

action of the T transforms GDA to another objects: $\langle 0|\bar{\psi}\left(-\frac{z}{2}\right)\gamma \cdot z\psi\left(\frac{z}{2}n\right)|-p',p\rangle$, describing the appearance of $h\bar{h}$ pair in the initial, rather than in the final state. Unfortunately, they, strictly speaking, cannot be accessed in the process of $h\bar{h}$-annihilation to the $\gamma^*\gamma$ pair, being the crossing version of DVCS and $h\bar{h}$ production, as the mass of virtual photon, which should provide the 'hard' scale, is constrained by the kinematics to be smaller than $s = W^2$, which is assumed to be the 'soft' parameter. At the same time, one may consider the $h\bar{h}$ (in reality, probably $p\bar{p}$) annihilation to the pair of two *real* photons at (moderately) large s as a crossing version of the large angle (real) Compton scattering, which is known to be described by GPD [13]. This may give access to the $p\bar{p}$ GDA, very difficult otherwise.

The relations (6) lead to the following symmetry properties for SPD and GDA

$$H(x,\xi) = H(x,-\xi); \Phi(z,\xi) = -\Phi(-z,-\xi), \tag{7}$$

The additional symmetry properties emerge if the pions are in the definite isospin state (where we assume that more general symmetry (6,7) also holds)

$$F^{I=0}(x,y) = -F^{I=0}(-x,y); G^{I=0}(x,y) = G^{I=0}(-x,y),$$
$$F^{I=1}(x,y) = F^{I=1}(-x,y); G^{I=1}(x,y) = -G^{I=1}(-x,y)$$
$$H^{I=0}(x,\xi) = -H^{I=0}(-x,\xi); \Phi^{I=0}(z,\xi) = -\Phi^{I=0}(-z,\xi) = -\Phi^{I=0}(z,-\xi)$$
$$H^{I=1}(x,\xi) = H^{I=1}(-x,\xi); \Phi^{I=1}(z,\xi) = \Phi^{I=1}(-z,\xi) = -\Phi^{I=1}(z,-\xi). \tag{8}$$

For simplicity, we shall limit ourselves for the time being to the case of DD $F(x,y)$ only, neglecting the Polyakov-Weiss [10] term, resulting from the function $G(x,y)$. This approximation is self-consistent in the case of $I = 1$ [10].

Double distributions from the inverse Radon transform

We now make the key observation, that the relation between SPD and DD is nothing else than a particular case of the Radon transform [14, 15, 16, 17]. As soon as all the possible lines crossing the compact region are considered, one is dealing with the mapping between functions of two variables (because each line is characterized just by two real numbers). As soon as the transformed function is continuous, it may be recovered by making use of the inverse Radon transform. This procedure is known to be the key ingredient of the numerous applications [16], covering, say, medicine, optics and geophysics. We are going to suggest its application in (non-perturbative) QCD.

As soon as Radon transform is a new instrument in this field, it makes sense to present briefly the derivation of its inversion, which is actually very simple. It is more convenient to present it by parameterizing the straight line in a slightly different way: by using the unit vector $\vec{\xi} = (cos\phi, sin\phi)$ orthogonal to it, instead of skewedness ξ, and its distance from the origin p instead of the argument of the skewed distribution z. The correspondence to (3) is obvious.

$$R(p, \vec{\xi}) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy f(x,y) \delta(p - \vec{x}\vec{\xi}), \qquad (9)$$

$$\xi = tg\phi, z = p/cos\phi; H(z, \xi) = R(p, \vec{\xi})|cos\phi|. \qquad (10)$$

The most convenient way to invert the transform is to use its relation to Fourier transform. Indeed, the Fourier transform of the function $f(\vec{x}) \equiv f(x,y)$ may be written in the form

$$F(\vec{q}) = \int d^2\vec{x}\, e^{i\vec{x}\vec{q}} f(\vec{x}) = \int_{-\infty}^{\infty} dt \delta(t - \vec{x}\vec{q}) \int d^2\vec{x}\, e^{i\vec{x}\vec{q}} f(\vec{x}) \qquad (11)$$

Here the integration is performed in the whole (x,y) plane, while the correct integration limits are provided by the zero value of the function f outside region (1). It is instructive to specify the direction of $\vec{q}$ by the unite vector $\vec{\xi}$, which would be shown in a moment to be an argument of the Radon transform

$$F(\vec{\xi}\lambda) = \int_{-\infty}^{\infty} dt \delta(t - \lambda\vec{x}\vec{\xi}) \int d^2\vec{x}\, e^{i\lambda\vec{x}\vec{\xi}} f(\vec{x}) =_{t \to \lambda t} \int_{-\infty}^{\infty} dt \delta(t - \vec{x}\vec{\xi}) \int d^2\vec{x}\, e^{i\lambda\vec{x}\vec{\xi}} f(\vec{x}) =$$

$$\int_{-\infty}^{\infty} dt e^{i\lambda t} \int d^2\vec{x} \delta(t - \vec{x}\vec{\xi}) f(\vec{x}) = \int_{-\infty}^{\infty} dt e^{i\lambda t} R(t, \vec{\xi}) \qquad (12)$$

As a result, the two-dimensional Fourier transform may be represented as a combination of one dimensional Fourier transform and Radon transform [*]. Consequently, inverting the two-dimensional Fourier transform in the polar coordinates and making use of (12), the function may be expressed through its Radon transform

$$f(\vec{x}) = \frac{1}{(2\pi)^2} \int d^2\vec{q}\, e^{-i\vec{x}\vec{q}} F(\vec{q}) = \frac{1}{(2\pi)^2} \int_0^{\infty} \lambda d\lambda \int_0^{2\pi} d\phi\, e^{-i\lambda\vec{x}\vec{\xi}} F(\lambda\vec{\xi}) =$$

$$\frac{1}{(2\pi)^2} \int_0^{\infty} \lambda d\lambda \int_0^{2\pi} d\phi\, e^{-i\lambda\vec{x}\vec{\xi}} \int_{-\infty}^{\infty} dp\, e^{i\lambda p} R(p, \vec{\xi}) \qquad (13)$$

This formula already expresses the function $f(x,y)$ in terms of its Radon transform and the rest of derivation consists of its simplification. The latter starts with the change of variables $\phi' = \phi, p' = p - \vec{x}\vec{\xi}$

$$f(\vec{x}) = \frac{1}{(2\pi)^2} \int_0^{\infty} \lambda d\lambda \int_{-\infty}^{\infty} dp'\, e^{i\lambda p'} \int_0^{2\pi} d\phi' R(p' + \vec{\xi}\vec{x}, \vec{\xi}) \equiv \qquad (14)$$

$$\frac{1}{2\pi} \int_0^{\infty} \lambda d\lambda \int_{-\infty}^{\infty} dp\, e^{i\lambda p} \bar{R}(p, \vec{x}).$$

Here $\bar{R}(p, \vec{x})$ is, obviously, the average of the Radon transform over all the straight lines, tangent to the circle of radius p and center in the actual point $\vec{x}$.

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[*]This is actually the core of the relation between DD and SPD, being, respectively, the two-dimensional and one-dimensional Fourier transforms of the same matrix element.

$$\bar{R}(p, \vec{x}) = \frac{1}{2\pi} \int_0^{2\pi} d\phi R(p + \vec{\xi}\vec{x}, \vec{\xi}). \qquad (15)$$

To present the inversion formula in its final form, one should use the integration by parts and substitute the formula for (one-dimensional) Fourier transform of the function $sign(\lambda)$

$$f(\vec{x}) = \frac{1}{4\pi} \int_{-\infty}^{\infty} sign(\lambda)\lambda d\lambda dp e^{i\lambda p} \bar{R}(p, \vec{x}) = \frac{i}{4\pi} \int_{-\infty}^{\infty} sign(\lambda) d\lambda dp e^{i\lambda p} \bar{R}'_p(p, \vec{x}) =$$

$$-\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dp}{p} \bar{R}'_p(p, \vec{x}) = -\frac{1}{\pi} \int_0^{\infty} \frac{dp}{p^2}(\bar{R}(p, \vec{x}) - \bar{R}(0, \vec{x})), \qquad (16)$$

where the last equality explores once more the integration by parts and the fact, that $\bar{R}(p, \vec{x}) = \bar{R}(-p, \vec{x})$. Recalling the definitions (3,15), one may express the double distri- butions directly in terms of skewed ones (c.f. [18]):

$$f(x, y) = -\frac{1}{2\pi^2} \int_0^{\infty} \frac{dp}{p^2} \int_0^{2\pi} d\phi |cos\phi|(H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2} \int_{-\infty}^{\infty} \frac{dz}{z^2} \int_{-\infty}^{\infty} d\xi(H(z + x + y\xi, \xi) - H(x + y\xi, \xi)). (17)$$

As one clearly see, to recover the double distribution one indeed should know the skewed distribution in an 'extended' region $-\infty < z, \xi < \infty$. The values $\xi < 0$ and , $|z|, |\xi| > 1$ are described by the crossing and symmetry relations. (5,7). it is easy to check by numerical integrations, that GPD obtained from various model DD [11] may be recovered with the reasonable accuracy.

As it often happened before in the history of the various Radon transform applications [16], some of its ingredients were rediscovered in the framework of GPD also. In fact, the basic inversion formula in the form of triple integral (similar to (13)) was introduced in the framework of GPD by A.V. Radyushkin [19] (see also [18]). Also, the polynomiality condition [11] is well known in the general framework of Radon transform as the Cavalieri conditions [15]. The term is related to the interesting geometric interpretation, so far not mentioned in the context of DD. Consider the first ($n = 0$) moment. It is nothing else than the volume of the figure, limited by the surface $z = f(\vec{x})$ and the plane x, y. At the same time, $R(\vec{\xi}, p)$ may be interpreted as a surface of the section of this figure by the plane, containing the line $p = \vec{\xi}\vec{x}$ and z axis. The first moment is then expressing the volume of the figure in terms of the surfaces of its intersection with the set of parallel planes. Obviously, the direction of these planes is unimportant, so that the moment does not depend on $\vec{\xi}$. The contact with the classical Cavalieri principle may be achieved by the particular choice of $f(\vec{x}) = 1$ everywhere in the region where it is defined. In that case one is dealing with the surface of this region, represented by the lengths of the set of parallel lines. Again it does not depend on their direction and $\vec{\xi}$.

Polyakov-Weiss terms and two-dimensional magneto-statics

Let us note that separation of the functions F and G is, generally speaking, impossible, as soon as only the information about leading twist SPD (GDA) is available. Indeed, using the

integration by parts and assuming that DD turn to zero at the boundary, one can absorb the factors $z \cdot P, z \cdot \Delta$ to the definitions of DD and rewrite (3) in the following way:

$$\langle p'|\bar{\psi}\left(-\frac{z}{2}\right)z \cdot \gamma\psi\left(\frac{z}{2}\right)|p\rangle = -i\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dye^{-ixPz-iy\Delta z/2}N(x,y) \quad (18)$$

$$N(x,y) = \frac{\partial F(x,y)}{\partial x} + \frac{\partial G(x,y)}{\partial y}, \quad (19)$$

One may absorb the factor $z \cdot P$ to the definition of the SPD in the analogous way. As a result, the 'effective' DD $N(x,y)$ is related by the Radon transform to the 'effective' SPD, which is just the derivative $\frac{\partial H(z,\xi)}{\partial z}$ of standard SPD. Consequently, it is only possible to recover the effective DD, while separation of the individual contributions from F and G is, generally speaking, impossible.

It is interesting, that this ambiguity is completely analogous to the one arising in the problem of the recovery of the electromagnetic vector potential from the magnetic field (with a zero total flux). Indeed, denoting $G(x,y) = A_x, F(x,y) = -A_y, N(x,y) = B_z$, we can write (19) as $\vec{B} = rot\vec{A}$. Consequently, the effective distribution $N(x,y)$ does not change when F and G undergo the following 'gauge transformation'

$$F(x,y) \rightarrow F(x,y) + \frac{\partial\alpha(x,y)}{\partial y},$$

$$G(x,y) \rightarrow G(x,y) - \frac{\partial\alpha(x,y)}{\partial x},$$

$$\alpha(x,y) = -\alpha(x,-y), \quad (20)$$

where the last equation follows from T- (or C-) invariance [6]. One may try to choose

$$\alpha(x,y) = \int^{x}dtG(t,y), \quad (21)$$

so that the G term is completely eliminated. In order to preserve the symmetry properties (8) after the gauge transformation, the boundary conditions in (21) should be chosen in a following way:

$$\alpha(x,y) = \frac{1}{2}\left(\int_{|y|-1}^{x}dtG(t,y) - \int_{x}^{1-|y|}dtG(t,y)\right). \quad (22)$$

However, these boundary conditions are the zero ones in the points $x = \pm(1-|y|)$, which in this case gurantees the preservation of zero boundary conditions for F and G, only if

$$\int_{|y|-1}^{1-|y|}dxG(x,y) = 0, \quad (23)$$

which is indeed true for the case of $I = 1$ due to the symmetry properties (8). Consequently. in this case, when function G is not required by the polynomiality [10], it may be completely eliminated by the gauge transformation. At the same time, for $I = 0$ it is, generally speaking, impossible to assume simultaneously the zero boundary conditions at $x = 1 - |y|$ and $x =$

$|y| - 1$ and eliminate G completely. However, it is possible to reduce $G(x, y)$ to the function of *one* variable:

$$G(x, y) \to d(y) = \frac{1}{2(1 - |y|)} \int_{|y|-1}^{1-|y|} dx G(x, y),\qquad(24)$$

by the following choice of the gauge:

$$\alpha(x, y) = \frac{1}{2}\left(\int_{|y|-1}^{x} dt G(t, y) - \int_{x}^{1-|y|} dt G(t, y) \right) - x d(y).\qquad(25)$$

Here the x-independent contribution to $G(x, y)$ is provided by the last term which guarantees the zero boundary condition, One can change in this boundary-correcting term the linear function to the step one,

$$\alpha_{PW}(x, y) = \frac{1}{2}\left(\int_{|y|-1}^{x} dt G(t, y) - \int_{x}^{1-|y|} dt G(t, y) - sign(x) D(y) \right)$$
$$= \theta(x < 0) \int_{|y|-1}^{x} dt G(t, y) - \theta(x > 0) \int_{x}^{1-|y|} dt G(t, y),\qquad(26)$$

where $D(y)$ determines the resulting expression for G which now resides at $x = 0$:

$$G(x, y) \to \delta(x) D(y) = \delta(x) \int_{|y|-1}^{1-|y|} dt G(t, y).\qquad(27)$$

The existence of such a gauge transformation justifies the original suggestion [10] to consider this special form of $G(x, y)$.

Discussion and Conclusions

We studied crossing properties of Generalized Parton Distributions and showed, that Radyushkin's Double Distributions express them in the most simple way. Namely, they simultaneously describe, up to t and W^2 dependence, the DVCS and two-photon production of hadron pairs. This allows, in principle, to recover them from the data in the both channels, making use of the inverse Radon transform, solving therefore a problem of Radon tomography in the field of non-perturbative QCD. We also considered the ambiguity due to the Polyakov-Weiss terms and showed, that it is completely analogous for the gauge ambiguity of the vector potential of the static two-dimensional magnetic field. The observable quantity is in this sense 'effective' DD $N(x, y)$, which may be decomposed to the the F and G structures, adopting the particular gauge condition.

The development of the method may, first of all, include the more refined versions of Radon transform (see e.g. numerous Refs. in [16]), which would allow to use the limited range of angles of Radon projections (limited range of ξ). In that case, one may essentially limit the required input, and in particular, avoid the use of both channels. Also, one might include the possibilities of the singularities of recovered functions [17]. Other natural development would be to include the twist 3 contributions [20]. Another interseting opportunity may be provided by the process $\gamma^* \gamma \to 3\pi$, the GDA [21] for which depend on the *two* light-cone

fractions. The correspondent 'Triple Distributions' would depend on three variables, so to recover them one should consider the tomography in three-deimensional space, which (like for any odd-dimensional space) is known [15, 16] to have the local form due to the Hyugens principle.

Moreover, the Radon transform might be useful in the more general context of quantum field theory. Especially interesting seems to be its application to the general problem of crossing invariance, as it allows to consider (instead of functions in the *different* regions of their variables) the different projections of the *common* function, as it was discussed here in the particular case of GPD.

Another interesting application may arise from the fact, that inverse Radon transform naturally recovers information about the interior of some region, which makes a contact to the famous holographic principle [22]. One could add here, that Radon transform technique is known for a years to be one of the main tools in the field of *optical* holography [16].

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